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"RECENT ADVANCES IN THE EDGE-FUNCTION METHOD 1979-80"

by

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Main advances of E.F.M. in 1979-80 dealt with are :

(1) Anisotropic thin plates,
(2) Free Vibrations of thin plates,
(3) Analysis of Bridge Slabs,
(4) Elliptical Cracks in a Prismoidal body.

abstracts, conclusions, and illustrative examples are given in this report.

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"RECENT ADVANCES IN THE EDGE-FUNCTION METHOD 1979-80"

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JULY 1980

Recent Advances in The Edge-Function Method

Summary

Major advances in the Edge-Function Method in 1979-80 were made in four main areas:

1. Anisotropic Thin Plates (J.J. Grannell and P.M. Quinlan).
2. Free Vibrations of Thin Plates (Richard Sheehy).
3. Analysis of Elastic Plates and Bridge Slabs (A.A. Ahmud).
4. Elliptical Cracks in a Prismoidal Body (P.M. Quinlan, J.J. Grannell, A.N. Atluri, J.E. Fitzgerald).

The principal investigator was actively involved in an advisory role with both R. Sheehy and A.A. Ahmed on their Ph.D. theses. The present report gives an abstract and conclusions in each of the above works. It also gives some examples from (2) and (4) to illustrate the range of E F.M. in Free Vibrations and in buried Elliptical Cracks in Fracture Mechanics.

These examples are the touchstone by which the present state of development and utility of E.F.M. can be judged. In all cases root mean square boundary residuals are calculated as a routine, thus providing a practical measure for judging the acceptability of the solution proferred. If the residuals are within the limits within which an engineer can specify the boundary conditions of the problem, then the corresponding Mathematical Model which E.F.M. offers is as "exact" a solution to the physical model, or problem, as can be obtained.

1. Grannell J.J. and Quinlan P.M.
"The Edge-Function Method for Thin Anisotropic Plate Bonding"
Vol. 80A No. 1 Proc. Roy. Irish Academy 1980

Abstract

The edge-function method is developed for the bending of thin anisotropic polygonal plates. The solution is represented as a superposition of asymptotic solutions of the plate bending equation in the neighbourhood of characteristic segments of the boundary. In the case of a polygonal plate these segments are edges and vertices. Interior polynomial solutions are included and transverse loadings are modelled using a Green's function approach. The boundary conditions are imposed using accelerated discrete least-squares Fourier interpolation over each of the edges. The accuracy of the model is assessed by computing the boundary residuals. A symmetry principle is developed for solving symmetric boundary value problems. The solution of three plate problems is presented together with details of accuracy and convergence.

Three Plate Problems studied

1. Orthotropic Cantilever Plate under uniform normal load
2. Orthotropic Skew Plate under uniform normal load. Plate fully fixed on all edges.
3. Anisotropic simply supported square plate under uniform normal load.

Conclusions

The boundary residuals, in each of the problems studied, exhibit a steadily decreasing pattern with increasing levels of truncation of the boundary identities. The values of the residuals as percentages of the maximum values of the corresponding quantity occurring over the plate are well within the limits to which the boundary conditions can practically be specified. The very rapid convergence of the fields in the case of the cantilever plate may be attributed to the use of a symmetrised representation, this being a consistent feature of the use of the symmetrisation principle. In the case of the fully fixed and simply supported plates, where the symmetrised representation was not used, the symmetry in the computed fields becomes more exact with increasing truncation level. The consistent preference shown by the solver routine for vertex functions as opposed to polar functions reinforces the expectations of accuracy and efficiency for the vertex functions.

The relatively small number of degrees of freedom required to obtain rapidly convergent solutions demonstrates the efficiency of the asymptotic field representation approach, especially in the case of the severe singular field behaviour exhibited in the case of the cantilever plate problem.

2. "The Edge-Function Method for Free Vibrations of Isotropic Plates"
Sheehy Richard. Ph.D. thesis University College Cork July 1979

Abstract

Previous research on the isotropic plate vibration problem is reviewed briefly. The Edge-Function Method for steady-state vibrations of polygonal plates is developed, and asymtotic expansions of the solution in the neighbourhood of a corner are obtained under general boundary conditions and named Vertex Functions.

The boundary conditions lead to a system of homogenous linear equations, the non-trivial solution of which yields the frequency spectrum. A brief outline of the computer program, VIBRAT, as developed for plate vibration problems is given and is included as an appendix. A representative selection of results obtained for a variety of plate geometries is presented and a critical comparison with published results is given. Hitherto unsolved problems for irregular quadrilaterals and polygons are presented as the touchstone with which to evaluate the Edge Function Method for Free Vibrations.

Illustrative Examples

Examples of

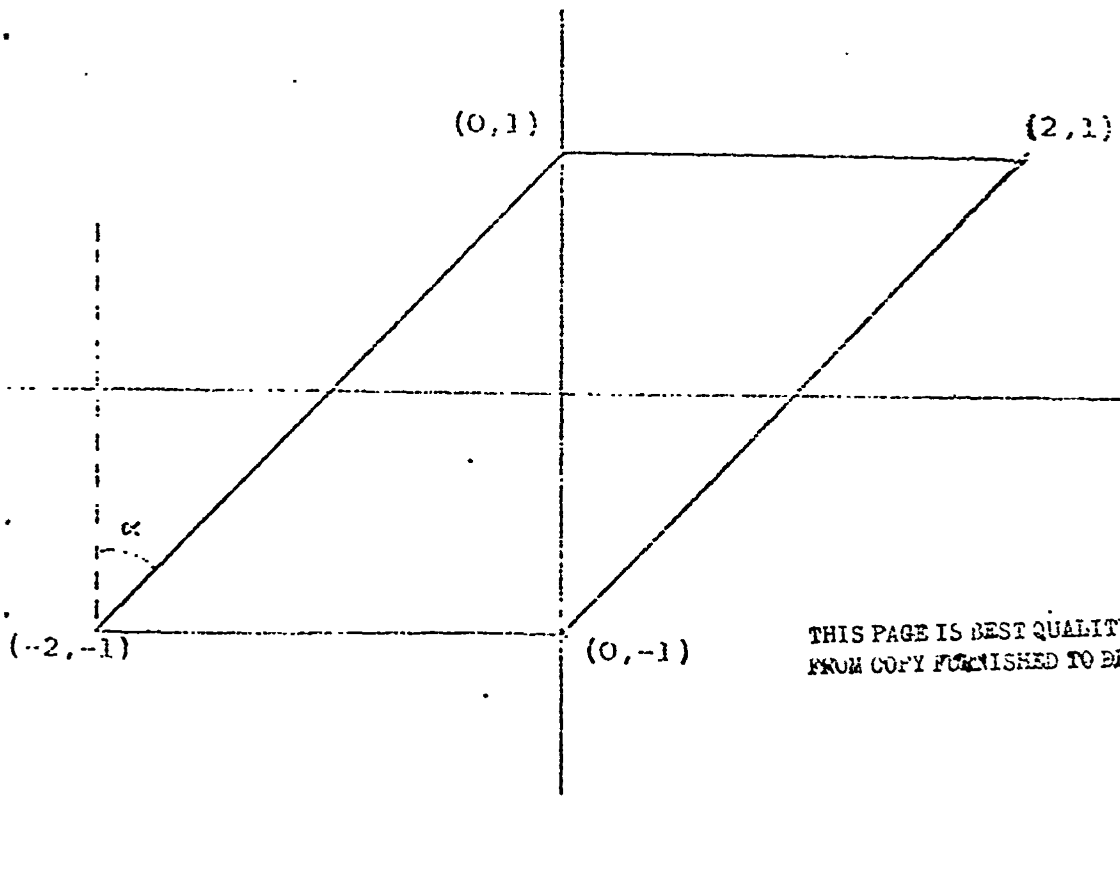
(3) Fully Fixed Rhombic Plate

(4) Quadrilateral Plate with Mixed Boundary Conditions

(5) Pentagonal Plate with Mixed Boundary Conditions,

together with Conclusions are attached from the above thesis to demonstrate the power of E.F.M. in Free Vibration problems.

EXAMPLE 3 – *Fully Fixed Rhombic Plate:*



The region in Fig.7.4 where all sides are clamped is next investigated. Results are compared to those of Kaul and Cadambe[37] using the Rayleigh-Ritz method. Frequency parameters are expressed in terms of $\omega a^2 \sqrt{\frac{\rho}{D}} \cos^2\alpha$ where α is the angle of skew.

Source	Mode and λ values		
	1	2	3
Kaul and Cadambe[37]	40.03	81.06	126.84
E.F.M. L=4	39.478	80.76183	125.984

TABLE 7.19

Percentage R.M.S.Boundary Residuals for the 1^{st} Frequency:

Function MT	Side j	% R.M.S. Residuals using Vertex Fns.
1	1	$.8264 \times 10^{-3}$
1	2	$.8731 \times 10^{-3}$
1	3	$.1137 \times 10^{-2}$
1	4	$.4347 \times 10^{-2}$
2	1	$.6272 \times 10^{-2}$
2	2	$.7360 \times 10^{-2}$
2	3	$.1737 \times 10^{-2}$
2	4	$.6818 \times 10^{-2}$

TABLE 7.20

Example 4 - Quadrilateral with Mixed Boundary Conditions

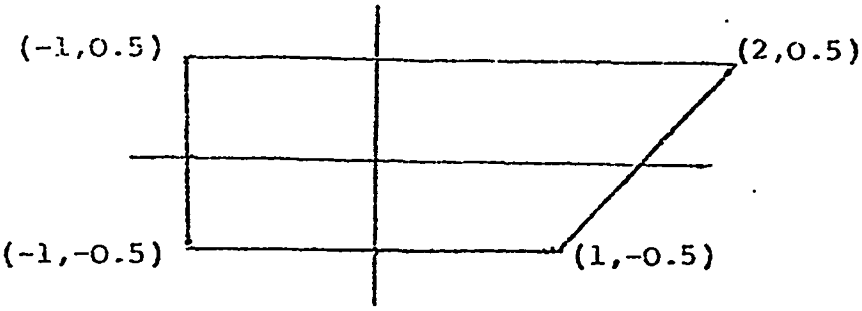


FIG.7.5

The region in Fig.7.4 with one side clamped and the other three simply supported was next investigated. Values for frequencies obtained by VIBRAT and presented in Table 7.21 are not compared with other investigators,since a search of the literature did not unearth such results.

	Mode and λ values		
	1	2	3
E.F.M. L=3	56.197	97.35	153.62;

TABLE 7.21

Percentage R.M.S. Boundary Residuals for the 1st Frequency:

Function MT	Side j	% R.M.S.Residuals using Vertex Fns.
1	1	0.9768×10^{-2}
1	2	0.9293×10^{-2}
1	3	0.7200×10^{-2}
1	4	0.8577×10^{-2}
2	1	0.2323×10^{-2}
3	2	0.1735×10^{-1}
3	3	0.5584×10^{-1}
3	4	0.1980×10^{-1}

TABLE 7.21a

Example 5 - Pentagon with Mixed Boundary Conditions

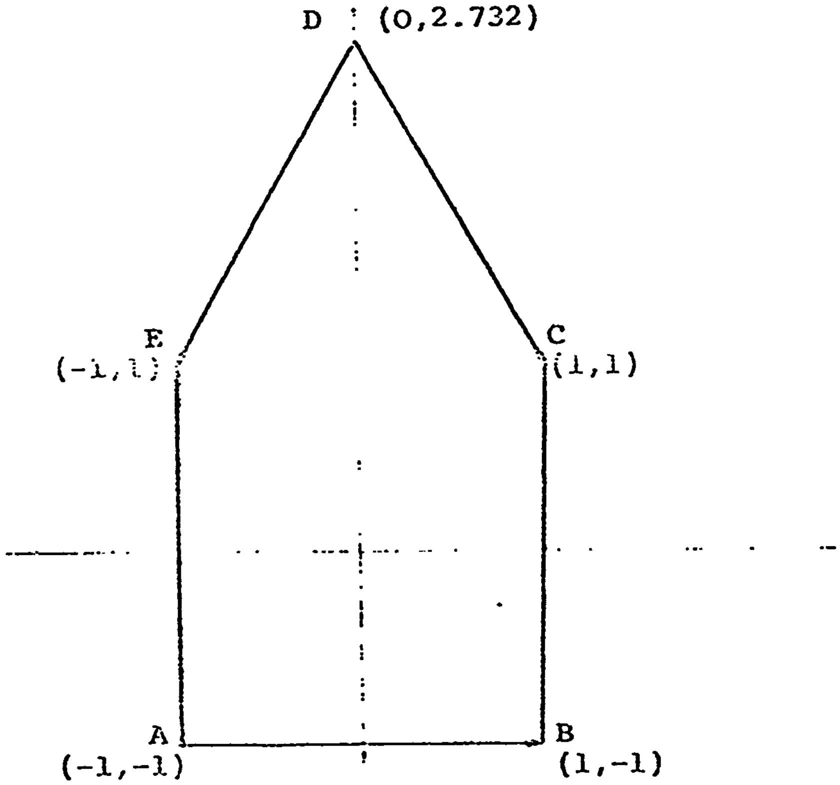


FIG.7.6

The region in Fig.7.6, where the side AB is clamped and the other four simply supported, is presented as a problem of far greater complexity than any that has been attempted by previous investigators. The results obtained by VIBRAT using truncation levels L=3,4 and 5 are given under in Table 7.22. The corresponding number of equations are 45, 55 and 65 respectively.

Source	Mode and λ values		
	1	2	3
E.F.M.			
L=3	13.097	27.621	34.2891
L=4	14.111	27.900	37.893
L=5	14.294	27.917	38.395

TABLE 7.22

The Edge-Function method with L=4, NIDEN=1 involves the use of 55 equations. The percentage R.M.S. boundary residuals for the first frequency using truncation levels L=4 and L=3 are given in Tables 7.23 and 7.24 respectively.

Function MT	Side j	% R.M.S. Residuals
1	1	.3745
1	2	.1528
1	3	.5202
1	4	.2863
1	5	.08379
2	1	.5175
3	2	.1206
3	3	.07968
3	4	.025255
3	5	.064173

TABLE 7.23

Function MT	Side j	% R.M.S. L = 3 Residuals
1	1	1.06665
1	2	0.95692
1	3	0.85699
1	4	1.2613
1	5	1.1892
2	1	0.6811
3	2	0.1047
3	3	0.7341
3	4	0.3224
3	5	0.1132

TABLE 7.24

CONCLUSION

The suite of programs included in Appendix B are written in general terms and allow for a "solution mix" of either scheme A - Edge-functions and Fractional-Edge functions - or scheme B - Edge-functions and Vertex functions.

Each solution obtained is an exact solution of a corresponding problem with corresponding boundary values as calculated from the solution vector, a measure of whose deviation from those prescribed is provided by the corresponding root-mean square boundary residuals. Accordingly, each solution provides a Mathematical Model for the given physical problem. The R.M.S. values provide a practical criterion for the engineer to judge the acceptability of the mathematical model. If the mathematical model lies within the scatter within which an engineer can specify the physical problem, then the mathematical model provides as "exact" a solution to the physical problem as can be obtained. A scatter of ±1% is used in this thesis.

The results in this thesis show:

(1) The frequencies with either "solution mix" are in excellent agreement with published results. In cases where results were not available there is excellent agreement between values obtained for various levels of harmonic truncation.

(2) Boundary Residuals Rectangular Regions: The R.M.S. boundary residuals for rectangular regions were quite acceptable using either "solution mix" with even as low a harmonic truncation level as L=2.

(3) R.M.S. Residuals - non Rectangular Regions: Scheme A

The combination of Edge Functions and Fractional Edge Functions did not produce boundary residuals in the case of non-rectangular regions within acceptable limits and, furthermore, the required symmetry in the associated mode shapes was not satisfactorily reproduced.

(4) Boundary Residuals - non Rectangular Regions: Scheme B

The introduction of Vertex functions of the form (4-13) reduced the boundary residuals by a factor between 10 and 100 from those obtained using Scheme A, and accordingly the number of harmonic sets, or the size of the matrix, required to obtain residuals within a scatter of $\pm$ 1%, can be correspondingly reduced.

(5) It can be concluded, from the array of examples presented, that Scheme B, the Edge-Function method involving the Vertex Function developed in this thesis, is both satisfactory and practical, involving relatively moderate computer requirements. It determines acceptable numerical values for the frequencies of free vibration and the associated mode shapes for general polygonal shaped plates with mixed boundary conditions.

The functions in Chapters III and IV may appear complicated but, in keeping with Edge-Function philosophy, the computer formulae (3-30) and (4-15) provide an easy means of controlling the resulting algebra. Accordingly, edge functions can almost be as readily applied in a computer program as, say, the trigonometric functions. The touchstone of any numerical method must be the ease with which the relevant theory can be programmed and the provision of a practical test to enable an engineer to decide whether or not to accept the results. The Edge-Function method for Vibration problems is presented for evaluation on the above criteria and especially on its success in solving quadrilateral and polygonal plate problems, as in examples (7-21) and (7-22) which, to the author's knowledge, have not been solved hitherto.

3. "Analysis of Elastic Plates using the Edge-Function Method"
Ahmed A.A., Ph.D. thesis Univ. of Dundee October 1979

Abstract

This thesis describes an investigation into the applicability of the edge-function method for the engineering analysis of plates and slab bridge decks. The specific objective is the assessment of accuracy attainable economically in evaluating deflections, bending moments and shears for the complex situations encountered by the bridge designers.

First, the development of the edge-function theory is presented for the analysis of thin plates and slabs with different planforms, types of loading and support conditions. The bending problem is formulated for both isotropic and orthotropic plates, and general solution forms are obtained. The technique is associated with solutions from all critical regions of the boundary and the presence of corner functions is found necessary to cater for singularities and to accelerate the convergence.

Two computer programs were developed to facilitate analysis of isotropic and orthotropic plates and slab bridge decks; they provide for the computation of column reactions as well as bending and twisting moments at any required station. A brief outline of the computer programs is given.

Numerical examples are given along with extensive comparisons with the results of other authors using other numerical methods and tests on models. The investigation shows excellent agreement and indicates that a generally accurate and inexpensive method of solution is now available. The advantages of this technique over the conventional domain type methods (finite element method and finite difference method), which require division of the whole plate into elements, lie in the much smaller size of the matrix to be solved, and it is thus more economical and suited for programming on medium size computers.

Recommendations for further development and application of the method to ther types of bridge deck are stated.

4. "The Edge-Function Method for Elliptical Cracks in a Prismoidal Body"
Quinlan P.M., Grannell J.J., Atluri S.N., Fitzgerald J.E.
Proceedings Second International Symposium on Innovative Numerical Methods in Engineering Science Montreal (1980).

Abstract

Stress fields are developed to model the behaviour of Elliptical cracks in a prismoidal body using confocal potential functions. Stresses and displacements are obtained in a form suitable for computation without having to introduce Jacobian Elliptic Function.

A special quadrature subroutine has been developed for Crack Functions and they are now incorporated into computer program PQDISK [5] for "3-Dimensional Stress Analysis using The Edge-Function Method".

Applications are made to a wide range of buried elliptic cracks, both large and small, under concentrated and distributed normal loadings. The effects of increasing the number of Crack Functions, and the corresponding boundary residuals, are studies and several illustrative examples are given.

INTRODUCTION

Since the publication by Shah and Kobayashi [1] in 1971 which developed the confocal potential functions, introduced by Segedin [2] for potential problems, to determine the Stress Intensity Factor for an Elliptical Crack in an infinite body under arbitrary normal loading, the extension of that work to finite bodies has presented a major challenge. A boundary integral solution was given by Cruse [3] in 1975 which was followed in 1977 by the hybrid element solution of Kathiresan [4] with a combination of finite and special crack tip elements.

However little progress appears to have been made in analytical methods using [1]. This is due mainly to the almost prohibitive algebraic difficulties in expressing in terms of Jacobian elliptic functions all the second and third derivatives of the stress potentials ϕ_{ij} required for the resulting boundary stresses, and difficulties in evaluation. Paper [1] obtains but one derivative, $\frac{\delta^2 \phi_{ij}}{\delta z^2}$, and that for only $i + j \leq 3$.

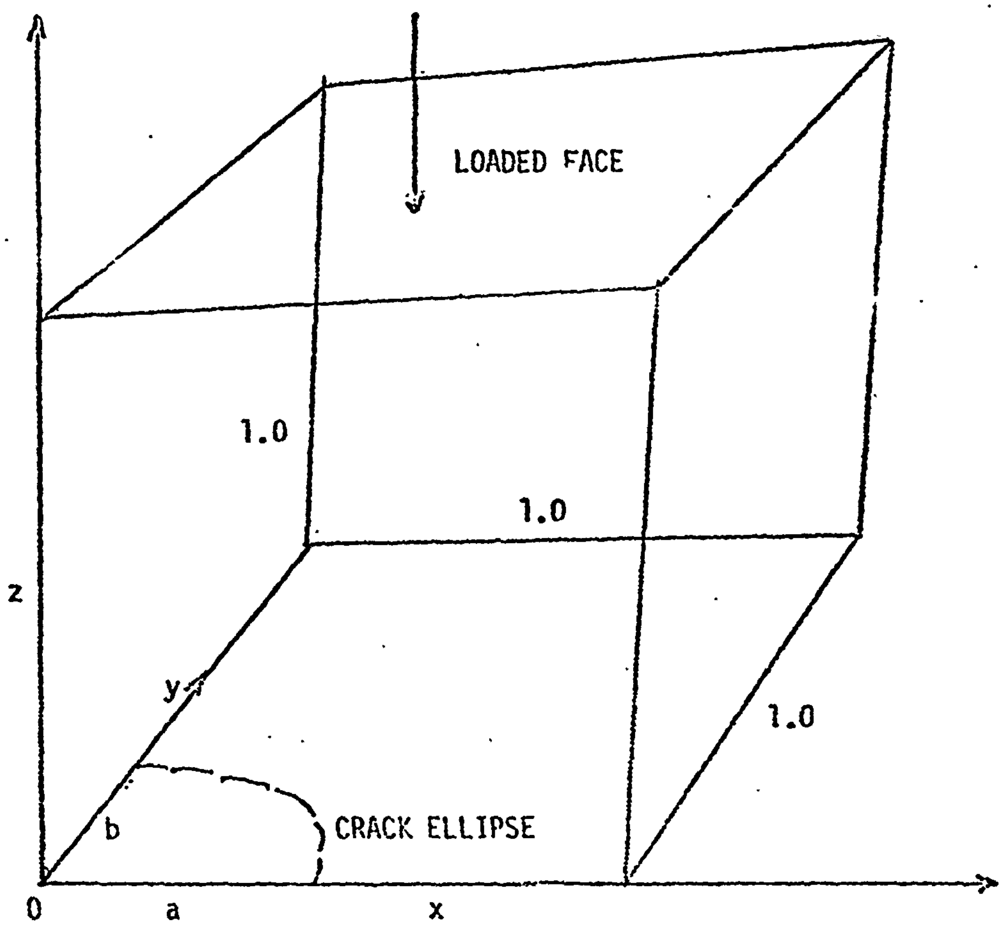
The present paper presents a straightforward calculation scheme for all the required derivatives of ϕ_{ij} without introducing any Jacobian elliptic functions and no particular difficulties are encountered with values of i and j in the range (0, 10). The resulting Elliptic Crack Functions have been included in the author's computer program PODISK [5] for Three Dimensional Stress Analysis in Prismoidal Bodies and numerical results are given for both small and large cracks. In sharp contrast to [3] and [4] large cracks can be handled as readily as small cracks. The computer times required are at least an order of mangitude less than those in [3] and [4]; stresses and displacements are readily calculated at any point, and each additional load case requires but a few percent of the time for the first case when all cases are processed together.

7. Numerical Examples

A Crack function [1.3]capability has been inserted in computer program PQDISK [5] which implements "The Edge Function Method for Prismoidal Bodies" and any required number from 1 to 15 of such functions, can be included in the solution scheme for modelling an elliptical crack.

The Edge Function Method is described in [6, 7, 8, 9, 11, 12], and space prevents any further elaboration here except to point out that each boundary condition is imposed on each face in either a continuous or a discrete least squares sense. Consequently the boundary condition of zero normal stress in the cracked ellipse is imposed in a least squares sense; the squares of the residuals in the cracked ellipse being minimised w r t the coefficients of the crack functions. The zero shear condition is provided by the built in symmetries of the functions.

A cube, 2 x 2 x 2, with a centrally located elliptical plane crack, loaded normally on the faces parallel to the crack plane as shown in Figure 2, is considered. Four different load types - (1) concentrated load (2) line load (3) patch load modelled by equivalent concentrated



OCTANT OF 3-SYMMETRY BODY

Figure 2

loads using the Bousinesque solution and (4) a linear stress field producing a uniform load, can be taken together in each job; the four loading cases processed together taking only about 15% more computer time than would be required to obtain a single solution on its own.

Four different problem sets are presented under to illustrate the use of the crack functions -

(a) Penny Shaped Crack

As given in Green and Sneddon [10] the fracture coefficient K_1^* for a penny shaped crack of radius a in an infinite solid in a uniform stress field p_0 normal to the plane of the crack is -

$$K_1^* = 2p_0\sqrt{a/\pi} \qquad (7.1)$$

The fracture coefficient K_1 was obtained for several values of the radius a and the ratio K_1/K_1^*, which measures the boundary effects on the fracture coefficient is given under, together with the root mean square of the residual stress, Δp, remaining on the crack ellipse

a	K_1/K_1^*	Δp
.2	1.02	$0.14*10^{-5}$
.4	1.11	$0.68*10^{-4}$
.6	1.39	$0.13*10^{-2}$
.8	2.34	$0.40*10^{-2}$

Table 2

(b) Small Elliptical Crack

The problems solved by Cruse [3] and Katiresan [4] obtained K_1 for relatively small ellipses where ab = .04 in Figure 2 with a/b = 1, 2, 3, 4 and under a uniform strees field. Program PODISK was run for these cases. Results are given in Table 3 under for K_1/K_1^* at 10 points on a quadrant of the

crack front corresponding to equal intervals in the elliptical angle θ for $\theta(0,\pi/2)$, and for the residual stress Δp on the crack ellipse. As given in Kathiresan [4], analogous to (7.1), K_1^* for all ellipse of semi-axes a and b is given by -

$$K_1^* = \frac{p_0\sqrt{\pi b}}{E(K)} \{\sin^2\theta + (\tfrac{b}{a})^2 \cos^2\theta\}^{\frac{1}{4}}, \quad a > b,$$

$$E(K) = \int_0^{\pi/2} \sqrt{1 - k^2\sin^2\psi}\, d\psi; \quad k^2 = 1 - b^2/a^2 \tag{7.2}$$

a/b	Ratio K_1/K_1^* at 10 points on Crack Front										Δp
1	1.02	Constant at all points									$0.14 * 10^{-5}$
2	.932	.952	1.00	1.07	1.14	1.20	1.25	1.29	1.31	1.32	$0.13 * 10^{-5}$
3	.828	.874	.976	1.09	1.19	1.28	1.35	1.39	1.42	1.43	$0.23 * 10^{-5}$
4	.746	.819	.961	1.11	1.22	1.32	1.39	1.45	1.48	1.49	$0.14 * 10^{-4}$

Table 3

The results all agree to within 2% with those given by Kathiresan [4]. They were obtained using 6 crack functions, together with 60 plane functions to model the boundary effects and a linear stress field. The computer time required for each ellipse is approximately six times that required to solve 70 simultaneous equations by ordinary Gaussian elimination and consequently is an order of magnitude less than was required in [3] or [4]. The equations took about five times as long to generate as to solve.

(c) Elongated Elliptical Crack

As a demonstration of the power of program PODISK examples (c) and (d) are presented. The first takes a very elongated crack with a = 0.8 and b = 0.05. It uses 120 plane functions to model the boundaries and 10 crack functions derived from [3.3] with the even values for i and j as under to provide the necessary symmetries -

i	0	0	2	0	2	4	0	2	4	6
j	0	2	0	4	2	0	6	4	2	0

(7.3)

Solutions are readily obtained, in the same job, using N_c = 1, 2, 3 ... 10 crack functions and for the four load types mentioned above. Table 4 shows the variations of the residual stress Δp on the crack plane for N_c crack functions and each load type, while Table 5 gives some illustrative K_1 factors at points 1, 3, 5, 7, 9 and 10 as in Figure 3, on a quadrant of the ellipse for concentrated and patch loads.

N_c	Concen. Load at (.2, .3)	Line Load from (0,.4) to (.5,.4)	Patch Load on 1x1	Uniform Stress
1	.411D 00	.172D 00	.151D-01	.275D-03
2	.407D 00	.168D 00	.150D-01	.272D-03
3	.269D 00	.171D-01	.505D-03	.184D-04
4	.266D 00	.169D-01	.505D-03	.182D-04
5	.266D 00	.169D-01	.506D-03	.181D-04
6	.112D-01	.438D-02	.362D-03	.125D-04
7	.112D-01	.436D-02	.362D-03	.125D-04
8	.111D-01	.434D-02	.358D-03	.120D-04
9	.110D-01	.434D-02	.352D-03	.117D-04
10	.921D-02	.433D-02	.271D-03	.749D-05

Table 4 Stress Residuals on a small Crack Plane (Effect of increasing the number of Crack Functions)

Concentrated Load		K_1 Factors at Points 1, 3, 5, 7, 9, 10					
N_c	1	.843	1.99	2.71	3.14	3.35	3.37
	2	.842	1.98	2.68	3.08	3.26	3.29
	3	.351	.994	1.92	2.98	3.71	3.81
	6	.392	1.04	1.89	2.93	3.72	3.84
	9	.392	1.04	1.89	2.93	3.72	3.83
Patch Load (Bousinesque)							
N_c	1	.399	.939	1.28	1.48	1.58	1.59
	2	.398	.939	1.28	1.48	1.58	1.59
	3	.380	.902	1.25	1.48	1.60	1.61
	5	.380	.902	1.25	1.48	1.60	1.61

Table 5 K_1 Fracture Coefficients (Crack Front Distribution)

Table 4 shows that six crack functions are required to reduce the residual stresses to 1% in the concentrated load case, while three functions reduce the residuals to less than .01% in the case of a patch load. The variations in the K_1 factors correspond to the residual stresses and no significant change occurs in the values after six and three crack functions respectively. The residual stresses on the surface of the cube were less than 3% of the applied loading and could be reduced further by increasing the number of plane functions used in the modelling.

(d) Large Elliptical Crack

Program PQDISK was run for a crack with a = 0.8, b = 0.7 using the same number of functions as in (c) and results analogous to those in Table 4 are given in Table 6.

N_c	Conc. Load at (.2, .3)	Line Load from (0, .4) to (.4, .4)	Patch Load on 1 x 1
1	.656D 00	.223D 00	.892D-01
2	.317D 00	.169D 00	.454D-01
3	.488D-01	.170D-01	.272D-02
4	.248D-01	.160D-01	.224D-02
5	.121D-01	.135D-01	.203D-02
6	.613D-02	.257D-02	.120D-02
7	.550D-02	.189D-02	.113D-02
8	.543D-02	.189D-02	.113D-02
9	.549D-02	.170D-02	.113D-02
10	.474D-02	.134D-02	.589D-03

Table 6 Stress Residuals on a Large Crack Plane (Effect of increasing the number of Crack Functions)

The K_1 factors corresponded to the residual stress pattern in a similar manner to the factors in Table 5. It is evident that five and three crack functions are required for concentrated and patch loads respectively to give K_1 factors to within 2%.

Work is progressing rapidly in adapting PQDISK to deal with surface and corners flaws and also to deal with fracture of Mode II and III since the corresponding crack functions can be generated from confocal harmonics (1.3). Results are expected in time for the Montreal meeting.

Displacements and stresses are readily calculated at any point in the body, and they are consistent with the boundary residuals reported in this section. Space limitations prevents their presentation in this paper.

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